

# 國立中央大學數學系

## 專題演講

主 講 人：Kasso Okoudjou教授(Tufts University)

演講題目：Independence of Weyl-Heisenberg coherent states: the HRT  
Conjecture

演講茶會：2026/05/26(星期二)10:30~11:00

茶會地點：中央大學鴻經館M306

演講時間：2026/05/26(星期二)11:00~12:00

演講地點：中央大學鴻經館M107

### Abstract :

In 1932, J. Von Neumann proposed that the coherent states

$$\mathcal{G}(g, 1, 1) = \{g_{n,k}(t) := e^{2\pi ikt} g(t - n) : k, n \in \mathbb{Z}\},$$

generated by the Gaussian  $g(t) = e^{-\pi t^2}$ , span a dense subspace of  $L^2(\cdot)$ . A decade later, D. Gabor independently formulated a similar idea in the context of communication theory.

These foundational observations gave rise to modern time–frequency analysis, which centers on coherent state systems of the form

$$\mathcal{G}(g, \Lambda) = \{g_\lambda(t) := e^{2\pi i\lambda_2 t} g(t - \lambda_1) : \lambda = (\lambda_1, \lambda_2) \in \Lambda\},$$

where  $g \in L^2(\mathbb{R})$  and  $\Lambda \subset \mathbb{R}^2$  is a discrete set.

Within this framework, C. Heil, J. Ramanathan, and P. Topiwala (1996) conjectured that for any nonzero square-integrable function  $g$  and any finite subset  $\Lambda = \{(a_k, b_k)\}_{k=1}^N \subset \mathbb{R}^2$ , the corresponding (finite) Weyl-Heisenberg system

$$\mathcal{G}(g, \Lambda) = \{e^{2\pi i b_k \cdot} g(\cdot - a_k)\}_{k=1}^N$$

is linearly independent. This statement, now known as the HRT Conjecture, remains a central open problem in time–frequency analysis despite sustained efforts over nearly three decades.

In this talk, I will first provide a historical overview of the conjecture, and then present recent progress on cases where the discrete set  $\Lambda$  is at most five.